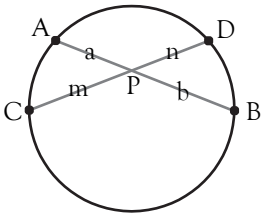
RELACIONES METRICAS EN LA CIRCUNFERENCIA

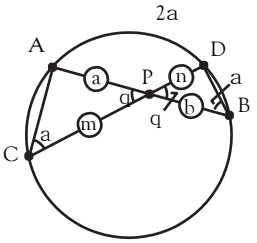
TEOREMA DE LAS CUERDAS

Si en una circunferencia se trazan dos cuerdas que se intersectan en un punto P, entonces los productos de los segmentos logrados en cada cuerda son iguales.



$$a \cdot b = m \cdot n$$

Demostración:



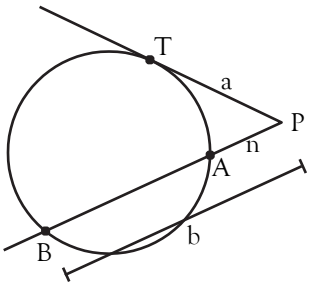
* DAPC ~ DPDB

$$\frac{a}{n} = \frac{m}{b}$$

$$a \cdot b = m \cdot n$$

TEOREMA DE LA TANGENTE

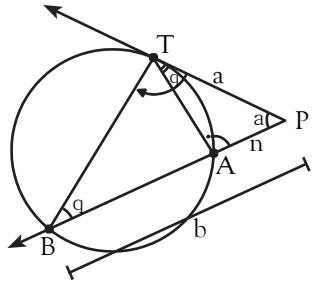
Si de un punto exterior a una circunferencia se traza una tangente y una secante, la tangente es media proporcional entre la secante y su parte externa.



Si T es punto de tangencia, entonces:

$$a^2 = b \cdot n$$

Demostración:



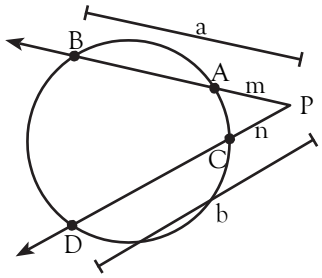
* DTPA ~ DBTP

$$\frac{a}{b} = \frac{n}{a}$$

$$a^2 = b \cdot n$$

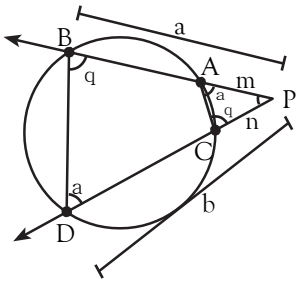
TEOREMA DE LAS SECANTES

Si desde un punto exterior a una circunferencia se trazan dos secantes, los productos de una de ellas y su parte externa son iguales.



$$a \cdot m = b \cdot n$$

Demostración:



* $\triangle$ DBAC es inscrito
$m\sphericalangle D = m\sphericalangle CAP$
$m\sphericalangle B = m\sphericalangle ACP$

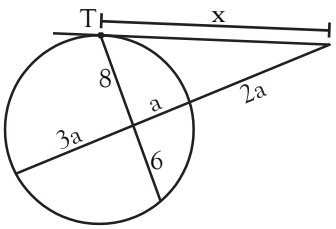
* DAPC ~ DBPD

$$\frac{n}{a} = \frac{m}{b}$$

$$b \cdot n = a \cdot m$$

Ejercicios Resueltos

1) Calcula x si T es punto de tangencia.



Resolución:

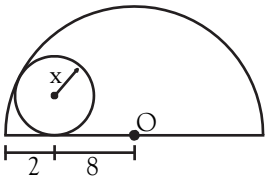
Por el teorema de las cuerdas:

$$(3a)(a) = (8)(6)$$
$$\Rightarrow \quad a = 4$$

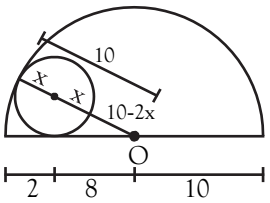
Por el teorema de la tangente:

$$x^2 = (2a)(6a)$$
$$\Rightarrow \quad x^2 = (8)(24)$$
$$\therefore \quad x = 8\sqrt{3}$$

2) Calcula x.



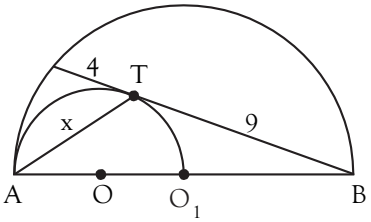
Resolución:



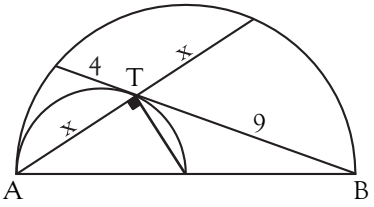
Por el teorema de la tangente:

$$8^2 = (10 - 2x)(10)$$
$$\Rightarrow \quad x = 3{,}6$$

3) Calcula x si T es punto de tangencia y O_1 es centro del diámetro $\overline{AB}$.



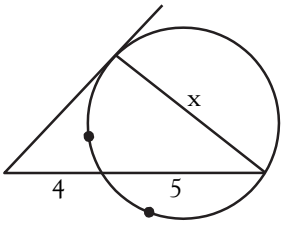
Resolución:



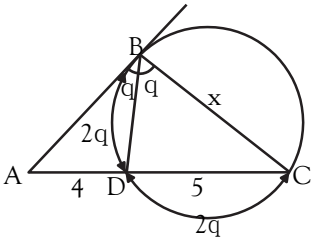
Por el teorema de las cuerdas:

$$(x)(x) = (4)(9)$$
$$x^2 = 36$$
$$\therefore \quad x = 6$$

4) En la figura, calcula x.



Resolución:



Teorema de la bisectriz

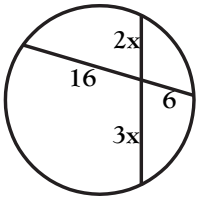
$$\frac{AB}{BC} = \frac{4}{5} \quad \Rightarrow \quad AB = (4/5)x$$

Por el teorema de la tangente:

$$(4/5)x^2 = 4(9)$$
$$\Rightarrow \quad 4/5x = 6$$
$$\therefore \quad x = 7{,}5$$

Resolviendo en clase

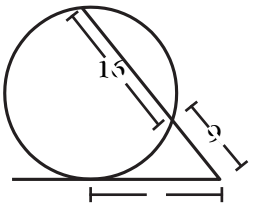
1 Calcula "x"



Resolución:

Rpta:

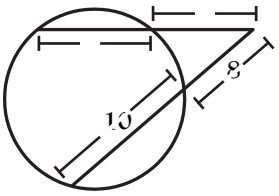
2 Calcula "x"



Resolución:

Rpta:

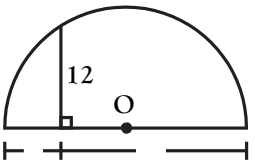
3 Calcula "x"



Resolución:

Rpta:

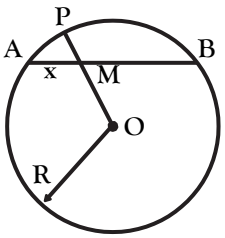
4 Calcula "x"



Resolución:

Rpta:

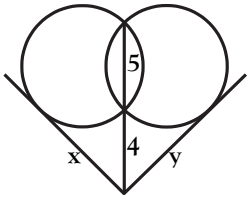
5 Calcula x si MB = 8, PM = 4 y OM = 5.



Resolución:

Rpta:

6 Calcula x + y.

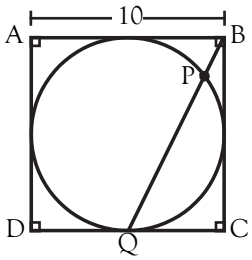


Resolución:

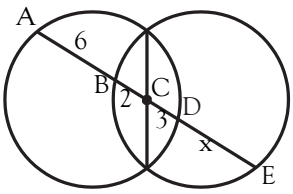
Rpta:

Ahora en tu cuaderno

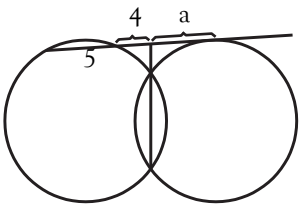
7. Calcula BP.



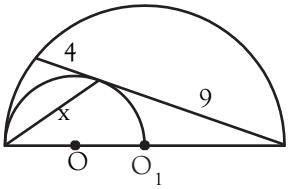
8. Calcula "x".



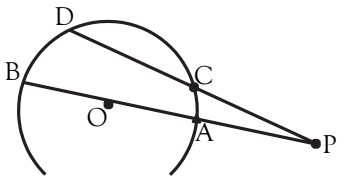
9. Halla "a".



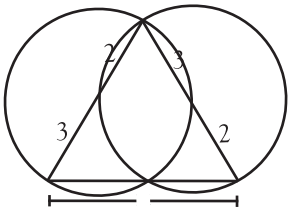
10. Halla "x".



11. Si "O" es centro de la circunferencia. Calcula su radio, además PC = 5, PA = 4 y CD = 3.

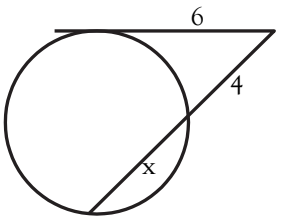


12. En la figura, calcula "x".



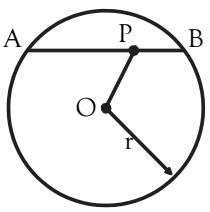
Para reforzar

1. Calcula "x"



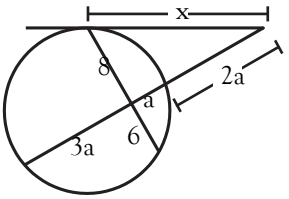
a) 9 b) 5 c) 4
d) 6 e) 8

2. Calcula r si AP = 6, PB = 4 y OP = 5.



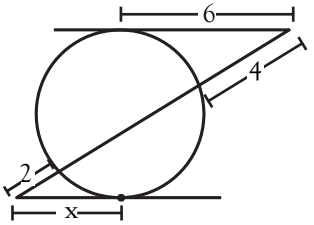
a) 7 b) 8 c) 9
d) 10 e) 6

3. Calcula "x".



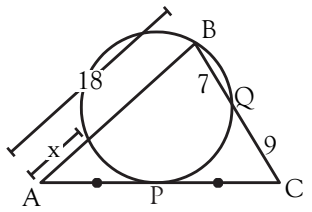
a) $4\sqrt{3}$ b) $\sqrt{3}$ c) 8
d) 12 e) $8\sqrt{3}$

4. Calcula "x".



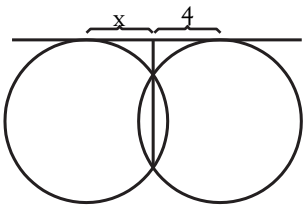
a) $\sqrt{7}$ b) $\sqrt{10}$ c) $\sqrt{14}$
d) $2\sqrt{7}$ e) 7

5. Calcula "x".



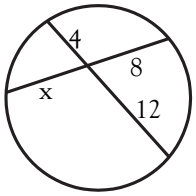
a) 9 b) 8 c) 7
d) 6 e) 5

6. Halla "x".



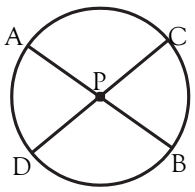
a) 2 b) 3 c) 4
d) 5 e) 8

7. Calcula "x".



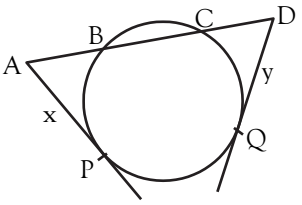
a) 2 b) 4 c) 6
d) 8 e) 10

8. Calcula "AB" si AP = 3, PC = 2 y PD = 6.



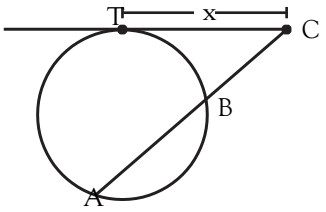
a) 4 b) 5 c) 6
d) 7 e) 8

9. Siendo "P" y "Q" puntos de tangencia; AB = 4, BC = 5 y CD = 3, calcula "xy".



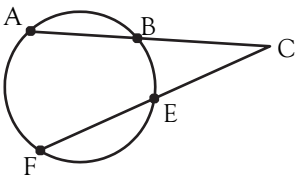
a) $6\sqrt{6}$ b) $12\sqrt{6}$ c) $8\sqrt{6}$
d) $10\sqrt{6}$ e) $\sqrt{6}$

10. Calcula "x" si AB=5 y BC=4. (T: punto de tangencia)



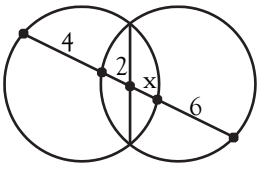
a) 4 b) 5 c) 6
d) 8 e) 9

11. Calcula "x" si AB=5, BC=4, FE=x y EC=3.



a) 8 b) 9 c) 10
d) 7 e) 6

12. Calcula el valor de "x".



a) 1 b) 1,5 c) 2
d) 2,5 e) 3