Triángulo Educativo

Geometría

4° SECUNDARIA

ANGULOS ENTRE PARALELAS

Introducción

En la vida diaria vemos constantemente rectas paralelas, como los pilares de una construcción o columnas, y diferentes tipos de figuras, como los puentes, etc.

Todo esto nos da la idea de rectas paralelas.

Paralelismo

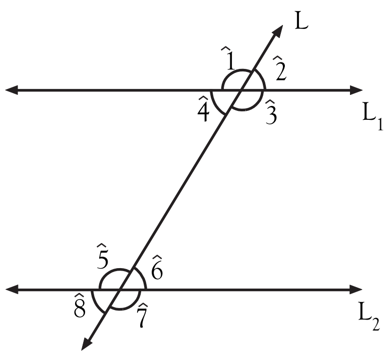
1. DEFINICIÓN

Dos rectas coplanares que no se intersecan son llamadas paralelas.

$$\overleftrightarrow{L_1} // \overleftrightarrow{L_2}$$

2. ÁNGULOS FORMADOS POR DOS RECTAS PARALELAS Y UNA RECTA SECANTE

Dada dos rectas $\overleftrightarrow{L_1}$ y $\overleftrightarrow{L_2}$ ($L_1 // L_2$), se dice que la recta L es una secante de ambas si las interseca en dos puntos diferentes.



Se cumple que:

- **Ángulos correspondientes** siempre son iguales.

$$\hat{1} = \hat{5};\ \hat{3} = \hat{7};\ \hat{2} = \hat{6};\ \hat{4} = \hat{8}$$

- **Ángulos alternos internos** siempre son iguales.

$$\hat{3} = \hat{5}\ ;\ \hat{4} = \hat{6}$$

- **Ángulos alternos externos** siempre son iguales.

$$\hat{2} = \hat{8}\ ;\ \hat{1} = \hat{7}$$

- **Ángulos conjugados internos** suman 180°.

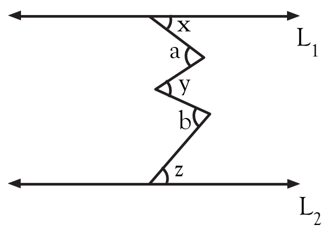
$$\hat{3} + \hat{6} = 180° \ ;\ \hat{4} + \hat{5} = 180°$$

- **Ángulos conjugados externos** suman 180°.

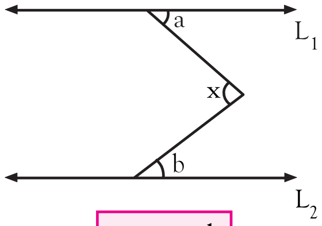
$$\hat{2} + \hat{7} = 180° \ ;\ \hat{1} + \hat{8} = 180°$$

3. PROPIEDADES

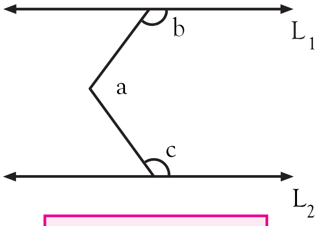
Si $L_1 // L_2 \Rightarrow$



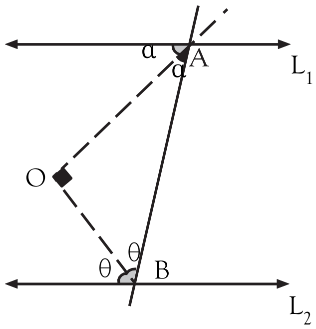
$$a + b = x + y + z$$



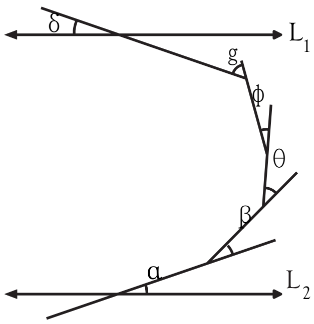
$x = a + b$



$360° = a + b + c$



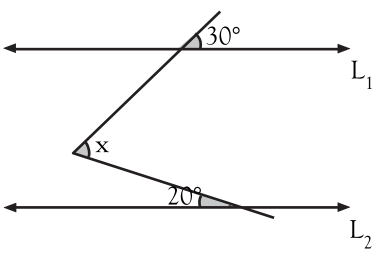
m∢AOB = 90°



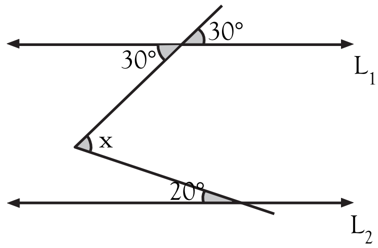
$\alpha + \beta + \theta + \phi + g + \delta = 180°$

Ejemplo 1:

Calcule "x".



Resolución:

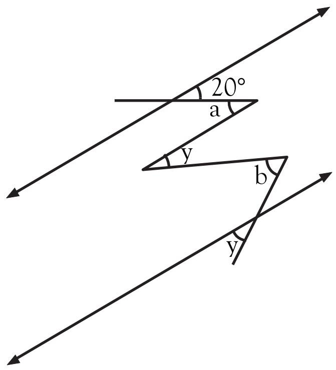


$x = 30° + 20°$

$x = 50°$

Ejemplo 2:

Calcule "y", si a + b = 60°.

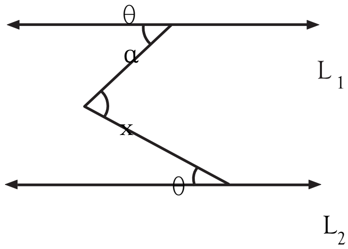


Resolución:

$$\begin{aligned} a + b &= y + y + 20° \\ 60° &= 2y + 20° \\ 40° &= 2y \\ 20° &= y \end{aligned}$$

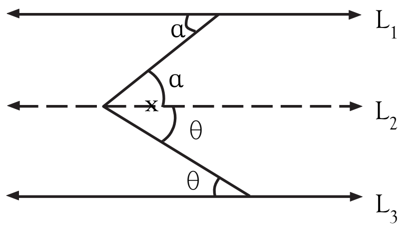
Demostración:

Si $L_1 // L_2 \Rightarrow x = \alpha \;\; +$



Resolución:

Por P trazamos una paralela a L_1 y L_2.

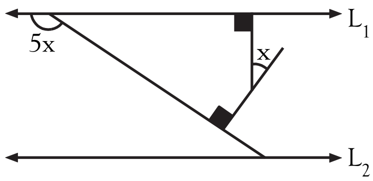


Luego por alternos internos:

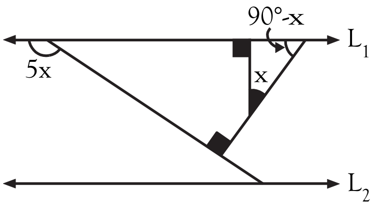
$\therefore x° = \alpha + \theta$

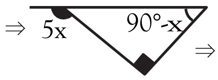
EJERCICIOS RESUELTOS

1. Calcule x, si: $L_1 // L_2$.



Resolución:



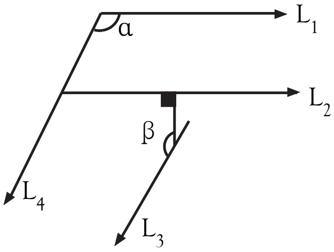


$\Rightarrow 90° - x + 90° = 5x$

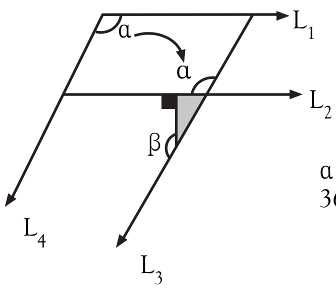
$180° = 6x$

$30° = x$

2. Calcule $\alpha + \beta$ si $L_1 // L_2 \wedge L_3 // L_4$.



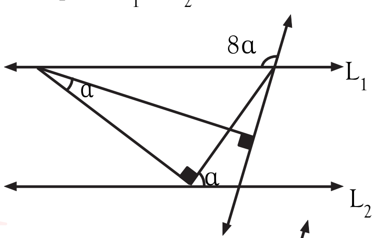
Resolución:



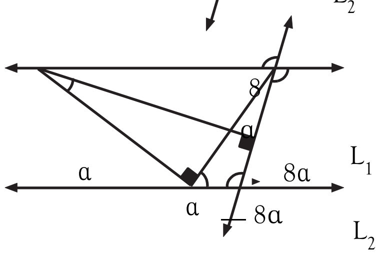
$\alpha + \beta + 90° = 360°$

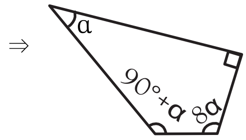
$\alpha + \beta = 270°$

3. Calcule $\alpha + \beta$ si: $L_1 // L_2$.



Resolución:

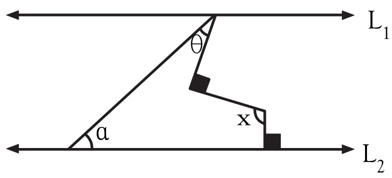




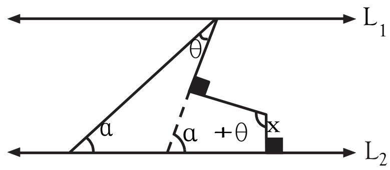
$\alpha + 90° + \alpha + 8\alpha + 90° = 360°$

$\alpha = 18°$

4. Calcule x si: $\alpha + \theta = 72°$ y $L_1 // L_2$.



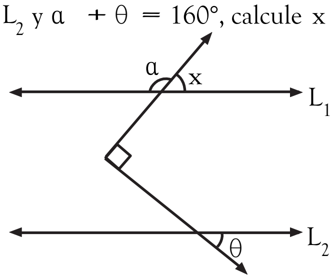
Resolución:



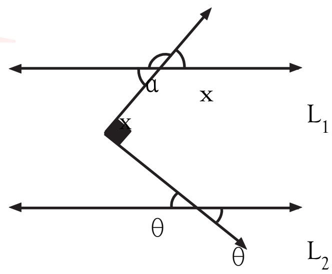
$\underbrace{\alpha + \theta}_{72°} + x + 90° + 90° = 360°$

$x = 108°$

5. Si $L_1 // L_2$ y $\alpha + \theta = 160°$, calcule x



Resolución:



$x + \alpha = 180°$

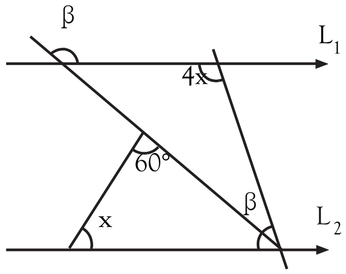
$\alpha + \theta = 90°$

$2\alpha + \underbrace{\alpha + \theta} = 270°$

$160° \quad x = 55°$

Resolviendo en clase

1 Calcule "x", si $\overleftrightarrow{L_1} // \overleftrightarrow{L_2}$.

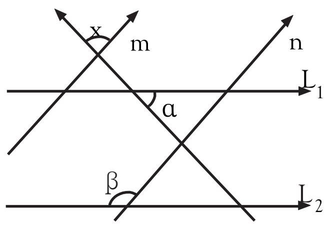


Resolución:

Rpta:

2 De la figura, β - α = 75°, $\overleftrightarrow{m} // \overleftrightarrow{n}$ y $\overleftrightarrow{L_1} // \overleftrightarrow{L_2}$.

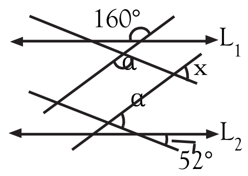
Calcule x.



~~Resolución:~~

Rpta:

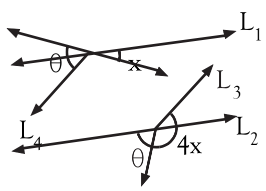
3 Calcule "x" si: $\overleftrightarrow{L_1} // \overleftrightarrow{L_2}$.



Resolución:

Rpta:

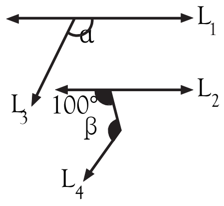
4 Calcule "x" si: $\overleftrightarrow{L_1} // \overleftrightarrow{L_2}$ y $\overleftrightarrow{L_3} // \overleftrightarrow{L_4}$.



~~Resolución:~~

Rpta:

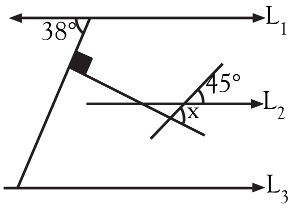
5 Calcule "α + β" si $\overleftrightarrow{L_1} // \overleftrightarrow{L_2}$ y $\overleftrightarrow{L_3} // \overleftrightarrow{L_4}$.



Resolución:

Rpta:

6 **Si: $L_1 // L_2 // L_3$, calcule «α».**

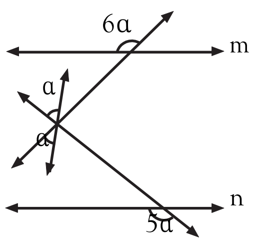


Resolución:

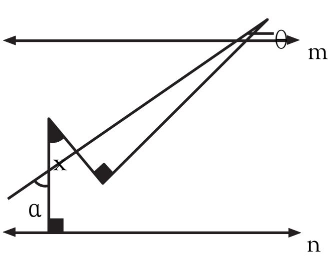
Rpta:

Ahora en tu cuaderno

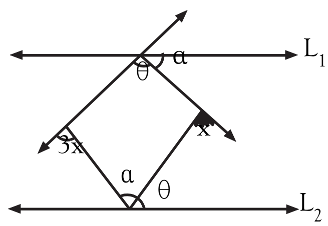
7. Si $\overleftrightarrow{m} // \overleftrightarrow{n}$, calcule "α".



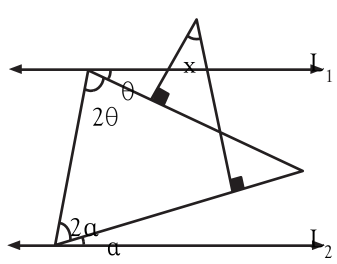
8. Calcule "x" si: $\overleftrightarrow{m} // \overleftrightarrow{n}$ y α – θ = 18°.



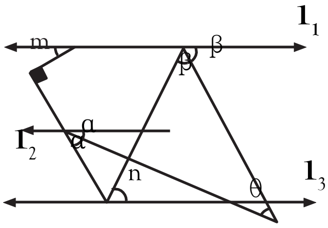
9. Calcule "x" si $\overleftrightarrow{L_1} // \overleftrightarrow{L_2}$.



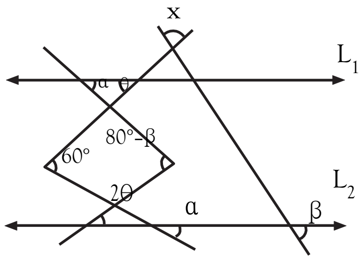
10. En la figura, calcule "x" si $L_1 // L_2$.



11. Si: $\overleftrightarrow{L_1} // \overleftrightarrow{L_2} // \overleftrightarrow{L_3}$ y m - n= 40°, calcule θ .

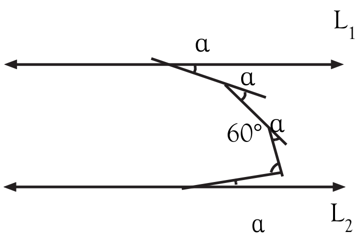


12. Si $\overleftrightarrow{L_1} // \overleftrightarrow{L_2}$, calcule x.



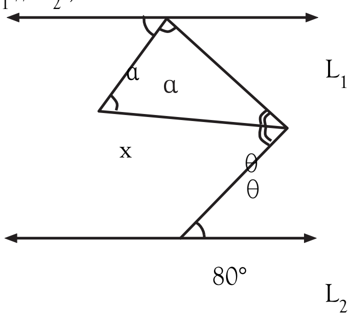
Para reforzar

1. Calcule "α" si: L_1 y L_2 son paralelas.



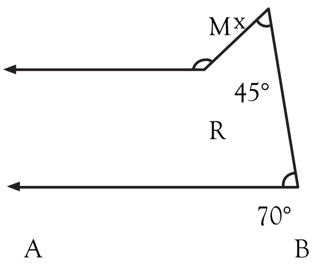
a) 10° b) 20° c) 15°
d) 25° e) 30°

2. Si: $\overleftrightarrow{L_1} // \overleftrightarrow{L_2}$, calcule x.



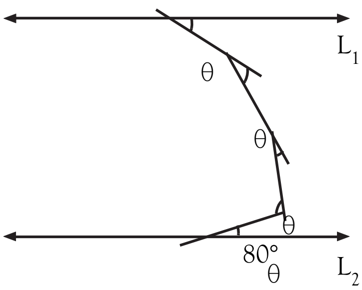
a) 40° b) 70° c) 10°
d) 50° e) 30°

3. Calcule "x" si: $\overline{AB} // \overline{MR}$



a) 120° b) 130° c) 115°
d) 100° e) 90°

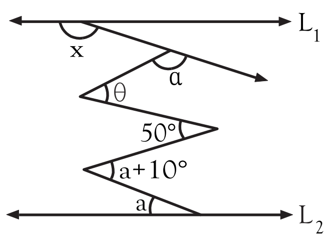
4. Calcule "θ " si L_1 y L_2 son paralelas.



a) 10° b) 60° c) 15°
d) 20° e) 40°

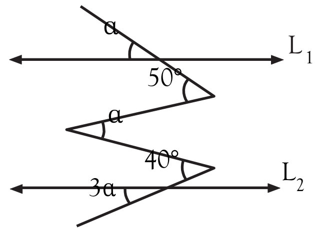
5. Calcule "x"; si: $\alpha + \theta = 170°$ y $// \overleftrightarrow{L_2}$.

L_1



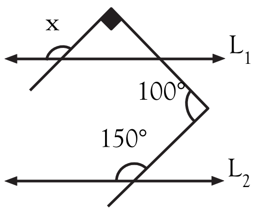
a) 110° b) 120° c) 130°
d) 135° e) 145°

6. Calcule "α" en la figura si: $\overleftrightarrow{L_1} // \overleftrightarrow{L_2}$.



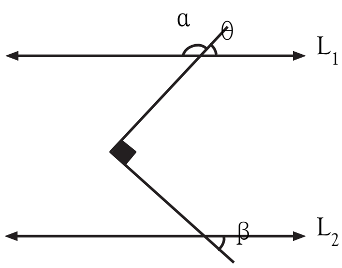
a) 12° b) 15° c) 16°
d) 18° e) 20°

7. Calcule "x" en la figura si: $\overleftrightarrow{L_1} // \overleftrightarrow{L_2}$.



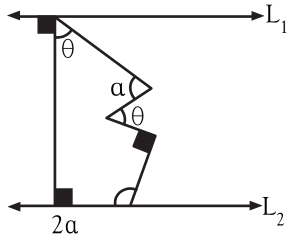
a) 100° b) 120° c) 130°
d) 150° e) 160°

8. Si: $\overleftrightarrow{L_1} // \overleftrightarrow{L_2}$ y $\alpha + \beta = 160°$, calcule θ.



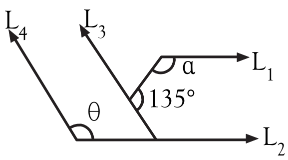
a) 40° b) 60° c) 35°
d) 20° e) 55°

9. Del gráfico, calcule "α".



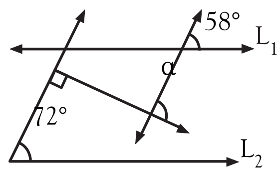
a) 60° b) 50° c) 45°
d) 40° e) 36°

10. Si: $\overleftrightarrow{L_1} // \overleftrightarrow{L_2}$ y $\overleftrightarrow{L_3} // \overleftrightarrow{L_4}$, calcule "$\alpha + \theta$".



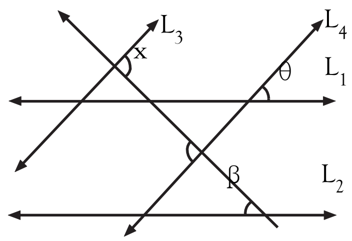
a) 125° b) 225° c) 325°
d) 220° e) 250°

11. Si: $\overleftrightarrow{L_1} // \overleftrightarrow{L_2}$, calcule "α".



a) 76° b) 72° c) 84°
d) 82° e) 90°

12. Si : $\overleftrightarrow{L_1} // \overleftrightarrow{L_2}$ y $\overleftrightarrow{L_3} // \overleftrightarrow{L_4}$ y $\theta + \beta = 120°$, calcule x.



a) 120° b) 150° c) 100°
d) 140° e) 90°